Geometry 1-2 Properties of Polygons	UNIT 5	Name: Teacher: Per:
My academic goal for this unit is		<ul> <li>Check for Understanding Key:</li> <li>Understanding at start of the unit</li> <li>Understanding after practice</li> <li>Understanding before unit test</li> </ul>

	LEARNING TARGETS	1	Hov unders	v is my standi	y ng?	Test Score	Retake?
5a	I can apply the polygon sum conjecture to determine unknown angle measures.	1	2	3	4		
5b	I can apply the exterior angle sum conjecture to determine unknown angles measures.	1	2	3	4		
5c	I can apply the properties of kites and trapezoids to determine unknown angle measures and segment lengths.	1	2	3	4		
5d	I can apply the properties of midsegments to determine unknown angle measures and segment lengths.	1	2	3	4		
5e	I can apply the properties of parallelograms to determine unknown angle measures and segment lengths.	1	2	3	4		
5f	I can prove the properties of quadrilaterals.	1	2	3	4		

What is an interior angle?

What is an exterior angle?

DP/1	CP/2	PR/3	HP/4
Developing Proficiency	Close to Proficient	Proficient	Highly Proficient
Not yet, Insufficient	Yes, but, Minimal	Yes, Satisfactory	WOW, Excellent
I can't do it and am not able to explain process or key points	I can sort of do it and am able to show process, but not able to identify/explain key math points	I can do it and able to both explain process and identify/explain math points	I'm great at doing it and am able to explain key math points accurately in a variety of problems

# Unit 5 Conjectures

Title	Conjecture	Diagram
Quadrilateral Sum Conjecture	The sum of the measures of the four angles in any quadrilateral is	
Pentagon Sum Conjecture	The sum of the measures of the five angles of any pentagon is	
Polygon Sum Conjecture	The sum of the measures of the n interior angles of an n-gon is	
Exterior Angle Sum Conjecture	For any polygon, the sum of the measures of a set of exterior angles is	
Equiangular Polygon Conjecture	You can find the measure of each interior angle of an equiangular n-gon by using either of these formulas	
Kite Angles Conjecture	The angles of a kite are	
Kite Diagonals Conjecture	The diagonals of a kite are	
Kite Diagonal Bisector Conjecture	The diagonals connecting the vertex angles of a kite is the of the other diagonal.	

# Unit 5 Conjectures

Title	Conjecture	Diagram
Kite Angle Bisector Conjecture	The angle of a kite are by a	
Trapezoid Consecutive Angles Conjecture	The consecutive angles between the bases of a trapezoid are	
Isosceles Trapezoid Conjecture	The base angles of an isosceles trapezoid are	
Isosceles Trapezoid Diagonals Conjecture	The diagonals of an isosceles trapezoid are	
Three Midsegments Conjecture	The three midsegments of a triangle divide it into	
Triangle Midsegment Conjecture	A midsegment of a triangle is to the third side and the length of 	
Trapezoid Midsegment Conjecture	The midsegment of a trapezoid is to the bases and is equal to	
Parallelogram Opposite Angles Conjecture	The opposite angles of a parallelogram are	

# Unit 5 Conjectures

Title	Conjecture	Diagram
Parallelogram Consecutive Angles Conjecture	The consecutive angles of a parallelogram are	
Parallelogram Opposite Sides Conjecture	The opposite side of a parallelogram are	
Parallelogram Diagonals Conjecture	The diagonals of a parallelogram	
Double-Edged Straightedge Conjecture	If two parallel lines are intersected by a second pair of parallel lines the same distance apart as the first pair, then the parallelogram formed is a	
Rhombus Diagonals Conjecture	The diagonals of a rhombus are and they	
Rhombus Angles Conjecture	The of a rhombus the angles of a rhombus.	
Rectangle Diagonals Conjecture	The diagonals of a rectangle are and	
Square Diagonals Conjecture	The diagonals of a square are, , and they	

## Notes

## Notes

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Find the interior angle sum for each polygon Roun	Date: Period	:
<ol> <li>regular 25-gon</li> </ol>	<ul><li>2) regular dodecagon</li></ul>	
3) regular 21-gon	4) regular 14-gon	
5)	6)	
7)	8)	

Find the measure of one interior angle in each regular polygon. Round your answer to the nearest tenth if necessary.

9) regular 16-gon

10) regular octagon

11) regular 14-gon



Find the measure of one exterior angle in each regular polygon. Round your answer to the nearest tenth if necessary.

17) regular 15-gon

18) regular pentagon

19) regular 14-gon

20) regular dodecagon









### Lesson 5.1 • Polygon Sum Conjecture

Name \_\_\_\_\_ Period \_\_\_\_ Date \_\_\_\_\_ In Exercises 1 and 2, find each lettered angle measure. 1.  $a = \___, b = \__, c = \__, d = \__, b = \__, c = \__, d = \__, e = \__, f = \____ d = \__, e = \__, f = \____ d = \__, e = \__, f = \____ d = \__, e = \__, f = \____ d = \__, e = \__, f = \_____ d = \__, e = \__, f = \_____, e = \__, e = \__, f = \_____, e = \__, e = \__, f = \____, e = \__, e = \__, f = \_____, e = \__, e = \__, f = \_____, e = \__, e = \__, f = \____, e = \__, e = \__, e = \__, f = \____, e = \__, e = \__, e = \__, f = \____, e = \__, e = \__, f = \___, e = \__, e$ 

- **3.** One exterior angle of a regular polygon measures 10°. What is the measure of each interior angle? How many sides does the polygon have?
- **4.** The sum of the measures of the interior angles of a regular polygon is 2340°. How many sides does the polygon have?
- **5.** *ABCD* is a square. *ABE* is an equilateral triangle.
- **6.** *ABCDE* is a regular pentagon. *ABFG* is a square.



- $x = \____{D}$
- 7. Use a protractor to draw pentagon *ABCDE* with  $m \angle A = 85^{\circ}$ ,  $m \angle B = 125^{\circ}$ ,  $m \angle C = 110^{\circ}$ , and  $m \angle D = 70^{\circ}$ . What is  $m \angle E$ ? Measure it, and check your work by calculating.

### Lesson 5.2 • Exterior Angles of a Polygon

Name	Period Date			
<b>1.</b> How many sides does a regular polygon have if each exterior angle measures 30°?	<b>2.</b> How many sides does a polygon have if the sum of the measures of the interior angles is 3960°?			

- **3.** If the sum of the measures of the interior angles of a polygon equals the sum of the measures of its exterior angles, how many sides does it have?
- **4.** If the sum of the measures of the interior angles of a polygon is twice the sum of its exterior angles, how many sides does it have?

In Exercises 5–7, find each lettered angle measure.



8. Find each lettered angle measure.



9. Construct an equiangular quadrilateral that is not regular.

### Lesson 5.3 • Kite and Trapezoid Properties



In Exercises 7 and 8, use the properties of kites and trapezoids to construct each figure. Use patty paper or a compass and a straightedge.

**7.** Construct an isosceles trapezoid given base  $\overline{AB}$ ,  $\angle B$ , and distance between bases *XY*.

**→** B

X •-

• Y

A •

**8.** Construct kite *ABCD* with  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{BD}$ .

 $\bullet B \quad B \bullet \bullet \bullet C \quad B \bullet \bullet \bullet D$ 

**9.** Write a paragraph or flowchart proof of the Converse of the Isosceles Trapezoid Conjecture. *Hint:* Draw  $\overline{AE}$  parallel to  $\overline{TP}$  with E on  $\overline{TR}$ .

**Given:** Trapezoid *TRAP* with  $\angle T \cong \angle R$ 

**Show:**  $\overline{TP} \cong \overline{RA}$ 



В

A •--

### Lesson 5.4 • Properties of Midsegments

Name	Period	Date
In Exercises 1–3, each figure sho	ows a midsegment.	
<b>1.</b> <i>a</i> =, <i>b</i> =,	<b>2.</b> $x = $ , $y = $ ,	<b>3.</b> $x = $ , $y = $ ,
c =	<i>z</i> =	<i>z</i> =
b a c 37°	14 $16$ $y$ $21$ $x$ $21$	$\begin{array}{c} & 41 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & $

**4.** *X*, *Y*, and *Z* are midpoints. Perimeter  $\triangle PQR = 132$ , RQ = 55, and PZ = 20.



**5.**  $\overline{MN}$  is the midsegment. Find the coordinates of *M* and *N*. Find the slopes of  $\overline{AB}$  and  $\overline{MN}$ .



**6.** Explain how to find the width of the lake from *A* to *B* using a tape measure, but without using a boat or getting your feet wet.



**7.** *M*, *N*, and *O* are midpoints. What type of quadrilateral is *AMNO*? How do you know? Give a flowchart proof showing that  $\triangle ONC \cong \triangle MBN$ .



**8.** Give a paragraph or flowchart proof.

**Given:**  $\triangle PQR$  with PD = DF = FH = HRand QE = EG = GI = IR**Show:**  $\overline{HI} \parallel \overline{FG} \parallel \overline{DE} \parallel \overline{PQ}$ 



Geometry 1-2	Name:	
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Parallelograms	Date:	Period:

Find the measurement indicated in each parallelogram.



















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### Lesson 5.5 • Properties of Parallelograms



**7.** Perimeter ABCD = 16x - 12. AD =\_\_\_\_\_



**8.** Ball B is struck at the same instant by two forces,  $\overline{F_1}$  and  $\overline{F_2}$ . Show the resultant force on the ball.



9. Find each lettered angle measure.





**10.** Construct a parallelogram with diagonals  $\overline{AC}$  and  $\overline{BD}$ . Is your parallelogram unique? If not, construct a different (noncongruent) parallelogram.

 $A \bullet \frown C \quad B \bullet \frown O$ 

### Lesson 5.6 • Properties of Special Parallelograms

Name	Period	Date
<b>1.</b> <i>PQRS</i> is a rectangle and $OS = 16$ .	<b>2.</b> <i>KLMN</i> is a square and $NM = 8$ .	<b>3.</b> <i>ABCD</i> is a rhombus, AD = 11, and $DO = 6$ .
<i>OQ</i> =	$m \angle OKL = \_$	$OB = \_\_\_$
$m \angle QRS = $	$m \angle MOL = \_$	$BC = \_$
$PR = \_$	Perimeter <i>KLMN</i> =	$m \angle AOD = $
p $Q$ $R$ $Q$		$A \xrightarrow{D} O \xrightarrow{C} C$
In Exercises 4–11, match each d	escription with <i>all</i> the terms that	fit it.
a Trapezoid b Is	posceles triangle - c Parallelogy	cam d Phombus

a. mapezoia	<b>D.</b> 1505celes trialigie	<b>c.</b> 1 al al	leiograffi	<b>u.</b> Ithohhbus
e. Kite	f. Rectangle	g. Squar	re	h. All quadrilaterals
4 Diagonals bise	ect each other.	5	Diagonals are	perpendicular.
<b>6.</b> Diagonals are	congruent.	7	_ Measures of in to 360°.	iterior angles sum
8 Opposite sides	s are congruent.	9	_ Opposite angle	es are congruent.
<b>10.</b> Both diagonal	s bisect angles.	11	_ Diagonals are bisectors of ea	perpendicular ch other.

In Exercises 12 and 13, graph the points and determine whether *ABCD* is a trapezoid, parallelogram, rectangle, or none of these.



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### **Lesson 5.7 • Proving Quadrilateral Properties**



## **Unit 5 • Challenge Problems**

### 1. (*Target 5a*)

On her Chapter 5 test, Ms. Donovan asked her students to find the measure of each interior angle of a regular 15-gon. Here are a few of the answers students gave:

0	0			0
2340°	156°	180°	24°	168°

- a. Tell which of the answers is correct and explain why it is correct.
- b. Choose two of the incorrect answers and explain the error the student may have made in finding the answer.

2. (*Target 5a & 5b*)

In Lesson 5.1, you found formulas for the sum of the interior angle measures of a regular polygon and for the measure of each interior angle. Now you will consider the "outside" angles of a regular polygon.

In the regular hexagon below, one "outside" angle is marked. The measure of this angle is the number of degrees in the rotation indicated by the arrow.

Find a formula for the sum of the measures of the "outside" angles of a regular n-gon and a formula for the measure of one "outside" angle. Show and explain all your work and simplify the formulas as much as possible.

## **Unit 5 • Challenge Problems**

### 3. (*Target 5a, 5b and 5c*)

A classical semicircular arch is really half of a regular polygon built with blocks whose faces are congruent isosceles trapezoids. For example, the inner arch in the diagram below is half of a regular 18-gon.



The angle measures of the isosceles trapezoids forming the arch depend on how many blocks are used to build the arch.

- **a.** If a semicircular arch has only three blocks, what are the angle measures of the isosceles trapezoids? If an arch has five blocks, what are the angle measures? Show all your work.
- **b.** Given the number of blocks, *b*, in the arch, find formulas for the measures of the trapezoid base angles. You should find two formulas—one for the "outer" base angles (those whose vertices are on the outside of the arch) and one for the "inner" base angles. Explain the reasoning you used to find the formulas.

4. (*Target 5c, 5d and 5e*)

In the game *Find the Oddball*, a player looks at four objects and determines which one doesn't belong with the others. For example, consider the four objects below.



You might say that object B doesn't belong because it has four sides, while the other shapes each have three. Or, you might say that object C doesn't belong because it is the only shape with a right angle. You may be able to find and explain other oddballs.

Create three different groups of four quadrilaterals, each with at least one shape that can be considered an oddball. For each group you create, identify every possible oddball and explain why it doesn't belong with the other objects.

## **Unit 5 • Challenge Problems**

### 5. (Targets 5c, 5e & 5f)

Consider the following points: *A*(1, 4), *B*(2, 12), *C*(9, 8).

- a. Graph the points. Add a fourth point *D* so that points *A*, *B*, *C*, and *D* are the vertices of a particular type of quadrilateral. Name the quadrilateral and use algebra to verify one of the properties of that type of quadrilateral. Show all your work.
- b. Find the coordinates of the point where the diagonals of quadrilateral *ABCD* intersect. Show all your work.

### 6. (*Target 5e*)

The modern art section of the Museum of Geometric Art is a large rectangular room. The museum directors want to build a wall in the center of the room to create more room for displaying art. The wall will be built so that it is parallel to two of the opposite sides and its ends are equally distant from the other two sides.

Once the center wall is in place, a path will be painted on the floor around it. The path will be created by connecting the midsegments of the triangles and trapezoids formed by connecting the ends of the center wall to the corners of the room. (See the diagram.)

**a.** If the room measures 80 ft by 100 ft and the wall is 70 ft long, how long will the path be? Does your answer depend on which sides the wall is parallel to? Explain.



**b.** Now generalize your results. If the room measures *a* feet by *b* feet and the center wall is *x* feet long, how long will the path around the wall be?

**7.** (See flowchart proof at bottom of page 102.)

#### 8. Flowchart Proof



#### LESSON 5.1 • Polygon Sum Conjecture



### LESSON 5.2 • Exterior Angles of a Polygon

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**1.** 
$$x = 30$$
**2.**  $x = 124^{\circ}, y = 56^{\circ}$ **3.**  $x = 64^{\circ}, y = 43^{\circ}$ **4.**  $x = 12^{\circ}, y = 49^{\circ}$ **5.**  $PS = 33$ **6.**  $a > 11$ 



#### 9. Possible answer:

**Paragraph proof:** Draw  $\overline{AE} \parallel \overline{PT}$  with E on  $\overline{TR}$ . TEAP is a parallelogram.  $\angle T \cong \angle AER$  because they are corresponding angles of parallel lines.  $\angle T \cong \angle R$  because it is given, so  $\angle AER \cong \angle R$ , because both are congruent to  $\angle T$ . Therefore,  $\triangle AER$  is isosceles by the Converse of the Isosceles Triangle Conjecture.  $\overline{TP} \cong \overline{EA}$  because they are opposite sides of a parallelogram and  $\overline{AR} \cong \overline{EA}$ because  $\triangle AER$  is isosceles. Therefore,  $\overline{TP} \cong \overline{RA}$ because both are congruent to  $\overline{EA}$ .

### LESSON 5.4 • Properties of Midsegments

**1.** 
$$a = 89^{\circ}, b = 54^{\circ}, c = 91^{\circ}$$

- **2.** x = 21, y = 7, z = 32
- **3.** x = 17, y = 11, z = 6.5
- **4.** Perimeter  $\triangle XYZ = 66$ , PQ = 37, ZX = 27.5
- **5.** M(12, 6), N(14.5, 2); slope  $\overline{AB} = -1.6,$ slope  $\overline{MN} = -1.6$
- **6.** Pick a point *P* from which *A* and *B* can be viewed over land. Measure *AP* and *BP* and find the midpoints *M* and *N*. AB = 2MN.



**7.** *AMNO* is a parallelogram. By the Triangle Midsegment Conjecture,  $\overline{ON} \parallel \overline{AM}$  and  $\overline{MN} \parallel \overline{AO}$ .

#### **Flowchart Proof**



8. Paragraph proof: Looking at  $\triangle FGR$ ,  $\overline{HI} \parallel \overline{FG}$  by the Triangle Midsegment Conjecture. Looking at  $\triangle PQR$ ,  $\overline{FG} \parallel \overline{PQ}$  for the same reason. Because  $\overline{FG} \parallel \overline{PQ}$ , quadrilateral FGQP is a trapezoid and  $\overline{DE}$  is the midsegment, so it is parallel to  $\overline{FG}$  and  $\overline{PQ}$ . Therefore,  $\overline{HI} \parallel \overline{FG} \parallel \overline{DE} \parallel \overline{PQ}$ .



#### LESSON 5.5 • Properties of Parallelograms

**1.** Perimeter ABCD = 82 cm **2.** AC = 22, BD = 14 **3.** AB = 16, BC = 7 **4.**  $a = 51^{\circ}, b = 48^{\circ}, c = 70^{\circ}$  **5.** AB = 35.5 **6.**  $a = 41^{\circ}, b = 86^{\circ}, c = 53^{\circ}$  **7.** AD = 75 **8. 8. 6. 7. 8. 7. 8. 7.**



**10.** No

**9.**  $a = 38^{\circ}, b = 142^{\circ}, c = 142^{\circ}, d = 38^{\circ}, e = 142^{\circ}, f = 38^{\circ}, g = 52^{\circ}, h = 12^{\circ}, i = 61^{\circ}, j = 81^{\circ}, k = 61^{\circ}$ 



#### LESSON 5.6 • Properties of Special Parallelograms

- **1.** OQ = 16,  $m \angle QRS = 90^{\circ}$ , PR = 32
- **2.**  $m \angle OKL = 45^\circ$ ,  $m \angle MOL = 90^\circ$ , perimeter KLMN = 32
- **3.**  $OB = 6, BC = 11, m \angle AOD = 90^{\circ}$



### LESSON 5.7 • Proving Quadrilateral Properties

- **1.** (See flowchart proof at bottom of page.)
- 2. Flowchart Proof



#### 3. Flowchart Proof





## Answers to Practice: Polygon Sum Conjecture

1) 4140°	2) 1800°	3) 3420°	4) 2160°
5) 900°	6) 1260°	7) 1620°	8) 1800°
9) 157.5°	10) 135°	11) 154.3°	12) 163.6°
13) 135°	14) 120°	15) 140°	16) 128.6°
17) 24°	18) 72°	19) 25.7°	20) 30°
21) 40°	22) 36°	23) 45°	24) 72°

## Answers to Parallelograms

1) 120°	2) 105°	3) 58°	4) 95°
5) 45°	6) 94°	7) 44°	8) 59°
9) 12	10) 11	11) 8	12) 9.1
13) 15	14) 32	15) 14.8	16) 19